લિબર્ટી પેપરસેટ

ધોરણ 12 : ગણિત

Full Solution

સમય : 3 કલાક **અસાઈનમેન્ટ પ્રશ્નપત્ર 10**

PART A

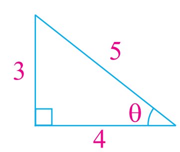
1. (D) **2.** (A) **3.** (A) **4.** (B) **5.** (A) **6.** (D) **7.** (B) **8.** (A) **9.** (A) **10.** (C) **11.** (A) **12.** (A) **13.** (C)
14. (C) **15.** (B) **16.** (C) **17.** (B) **18.** (B) **19.** (C) **20.** (B) **21.** (B) **22.** (A) **23.** (B) **24.** (B) **25.** (C)
26. (A) **27.** (B) **28.** (D) **29.** (C) **30.** (A) **31.** (C) **32.** (C) **33.** (A) **34.** (A) **35.** (A) **36.** (B) **37.** (B)
38. (B) **39.** (B) **40.** (C) **41.** (C) **42.** (A) **43.** (B) **44.** (A) **45.** (C) **46.** (A) **47.** (A) **48.** (B) **49.** (C)
50. (B)

PART B

વિભાગ-A

1.

ડા.બા. $= 2\ sin^{-1}\frac{3}{5}$



$sin^{-1}\frac{3}{5} = \theta$

$\therefore \quad sin\ \theta = \frac{3}{5}$

અહીં, $cos\ \theta = \frac{4}{5}$, $tan\ \theta = \frac{3}{4}$

હવે, $2\ sin\ \frac{3}{5} = 2\theta$

$$tan\ 2\theta = \frac{2\ tan\ \theta}{1 - tan^2\theta} = \frac{2\left(\frac{3}{4}\right)}{1 - \frac{9}{16}}$$

$$\therefore \quad tan\ 2\theta = \frac{\frac{3}{2}}{\frac{7}{16}}$$

$$\therefore \quad tan\ 2\theta = \frac{24}{7}$$

$$\therefore \quad 2\theta = tan^{-1}\left(\frac{24}{7}\right)$$

$$\therefore \quad 2\ sin^{-1}\frac{3}{5} = tan^{-1}\frac{24}{7}$$

2.

ડા.બા. $= cot^{-1}\left[\frac{\sqrt{1+sinx}+\sqrt{1-sinx}}{\sqrt{1+sinx}-\sqrt{1-sinx}}\right]$

$$= cot^{-1}\left[\frac{\sqrt{cos^2\frac{x}{2}+sin^2\frac{x}{2}+2sin\frac{x}{2}\cdot cos\frac{x}{2}}+\sqrt{cos^2\frac{x}{2}+sin^2\frac{x}{2}-2sin\frac{x}{2}\cdot cos\frac{x}{2}}}{\sqrt{cos^2\frac{x}{2}+sin^2\frac{x}{2}+2sin\frac{x}{2}\cdot cos\frac{x}{2}}-\sqrt{cos^2\frac{x}{2}+sin^2\frac{x}{2}-2sin\frac{x}{2}\cdot cos\frac{x}{2}}}\right]$$

$$= cot^{-1}\left[\frac{\sqrt{\left(cos\frac{x}{2}+sin\frac{x}{2}\right)^2}+\sqrt{\left(cos\frac{x}{2}-sin\frac{x}{2}\right)^2}}{\sqrt{\left(cos\frac{x}{2}+sin\frac{x}{2}\right)^2}-\sqrt{\left(cos\frac{x}{2}-sin\frac{x}{2}\right)^2}}\right]$$

$$= cot^{-1}\left[\frac{cos\frac{x}{2}+sin\frac{x}{2}+cos\frac{x}{2}-sin\frac{x}{2}}{cos\frac{x}{2}+sin\frac{x}{2}-\left(cos\frac{x}{2}-sin\frac{x}{2}\right)}\right]$$

$$\left\{\begin{array}{l} 0 < x < \frac{\pi}{4} \Rightarrow 0 < \frac{x}{2} < \frac{\pi}{8} \\ \Rightarrow cos\frac{x}{2} > sin\frac{x}{2} \\ \Rightarrow cos\frac{x}{2} - sin\frac{x}{2} > 0 \\ \Rightarrow \left|cos\frac{x}{2} - sin\frac{x}{2}\right| = cos\frac{x}{2} - sin\frac{x}{2} \\ \Rightarrow \frac{x}{2} \in \left(0, \frac{\pi}{8}\right) \subset (0, \pi) \end{array}\right\}$$

$$= cot^{-1}\left[\frac{2cos\frac{x}{2}}{2sin\frac{x}{2}}\right]$$

$$= cot^{-1}\left(cot\ \frac{x}{2}\right)$$

$$= \frac{x}{2} \quad \left(\because\ 0 < x < \frac{\pi}{4} \Rightarrow 0 < \frac{x}{2} < \frac{\pi}{8}\right)$$

= જ.બા.

અથવા

ડા.બા. $= cot^{-1}\left[\dfrac{\sqrt{1+sinx}+\sqrt{1-sinx}}{\sqrt{1+sinx}-\sqrt{1-sinx}}\right]$

$= cot^{-1}\left[\dfrac{\sqrt{\dfrac{1+sinx}{1+sinx}}+\sqrt{\dfrac{1-sinx}{1+sinx}}}{\sqrt{\dfrac{1+sinx}{1+sinx}}-\sqrt{\dfrac{1-sinx}{1+sinx}}}\right]$

($\because$ અંશ અને છેદને $\sqrt{1+sinx}$ વડે ભાગતાં)

$= cot^{-1}\left[\dfrac{1+\sqrt{\dfrac{1-cos\left(\frac{\pi}{2}-x\right)}{1+cos\left(\frac{\pi}{2}-x\right)}}}{1-\sqrt{\dfrac{1-cos\left(\frac{\pi}{2}-x\right)}{1+cos\left(\frac{\pi}{2}-x\right)}}}\right]$

$= cot^{-1}\left[\dfrac{1+\sqrt{tan^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}}{1-\sqrt{tan^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}}\right]$

$= cot^{-1}\left[\dfrac{1+\left|tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right|}{1-\left|tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right|}\right]$

$= cot^{-1}\left[\dfrac{1+tan\left(\frac{\pi}{4}-\frac{x}{2}\right)}{1-tan\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right]$

$= tan^{-1}\left[\dfrac{1-tan\left(\frac{\pi}{4}-\frac{x}{2}\right)}{1+tan\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right]$

$0 < x < \frac{\pi}{4}$

$\therefore 0 < \frac{x}{2} < \frac{\pi}{8}$

$\therefore 0 > -\frac{x}{2} > -\frac{\pi}{8}$

$\therefore \frac{\pi}{4} > \frac{\pi}{4} - \frac{x}{2} > \frac{\pi}{8}$

$\therefore tan\left(\frac{\pi}{4}-\frac{x}{2}\right) > 0$

$= tan^{-1}(1) - tan^{-1}\left(tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)$

$= \frac{\pi}{4} - \left(\frac{\pi}{4}-\frac{x}{2}\right)$ $\left(\because \left(\frac{\pi}{4}-\frac{x}{2}\right) \in \left(\frac{\pi}{8}, \frac{\pi}{4}\right) \subset \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right)$

$= \frac{x}{2}$ = જ.બા.

3.

f એ $x = 5$ આગળ સતત છે.

$\therefore \lim_{x\to 5^+} f(x) = \lim_{x\to 5^-} f(x) = f(5)$

$\therefore \lim_{x\to 5^+} (3x - 5) = \lim_{x\to 5^-} (kx + 1)$

$\left(\begin{array}{l}\because x\to 5^+\\ \Rightarrow x>5\\ \Rightarrow f(x)=3x-5\end{array}\right)$ $\left(\begin{array}{l}\because x\to 5^-\\ \Rightarrow x<5\\ \Rightarrow f(x)=kx+1\end{array}\right)$

$\therefore 3(5) - 5 = 5k + 1$

$\therefore 10 = 5k + 1$

$\therefore 5k = 9$

$\therefore k = \frac{9}{5}$

4.

$I = \int \dfrac{3x}{1+2x^4}\,dx$

$= \int \dfrac{3x}{(1)^2+\left(\sqrt{2}x^2\right)^2}\,dx$

અહીં, $\sqrt{2}\,x^2 = t$ આદેશ લેતાં,

$\therefore 2\sqrt{2}\,x\,dx = dt$

$\therefore x \cdot dx = \dfrac{dt}{2\sqrt{2}}$

$= \int \dfrac{3}{(1)^2+(t)^2}\,\dfrac{dt}{2\sqrt{2}}$

$= \dfrac{3}{2\sqrt{2}}\int \dfrac{dt}{(1)^2+(t)^2}$

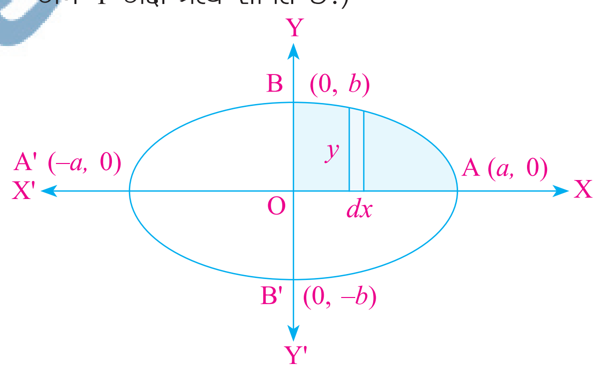
$= \dfrac{3}{2\sqrt{2}}\,tan^{-1}(t) + c$

$I = \dfrac{3}{2\sqrt{2}}\,tan^{-1}(\sqrt{2}\,x^2) + c$

5.

રીત 1 :

આકૃતિમાં દર્શાવ્યા પ્રમાણે ઉપવલય દ્વારા આવૃત્ત પ્રદેશ ABA'B'A નું ક્ષેત્રફળ = 4 × (આપેલ વક્ર, રેખાઓ $x = 0$, $x = a$ અને X-અક્ષ દ્વારા આવૃત્ત પ્રથમ ચરણમાં આવેલ પ્રદેશ AOBA નું ક્ષેત્રફળ). (ઉપવલય એ X-અક્ષ અને Y-અક્ષ પ્રત્યે સંમિત છે.)



માંગેલ ક્ષેત્રફળ $= 4\int_0^a y$ (શિરોલંબ પટ્ટીઓ લેતાં)

હવે, $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$. આથી $y=\pm\dfrac{b}{a}\sqrt{a^2-x^2}$

પરંતુ, પ્રદેશ AOBA પ્રથમ ચરણમાં આવેલો હોવાથી, y ને ધન લઈશું.

આથી માંગેલ ક્ષેત્રફળ,

$= 4\int_0^a \dfrac{b}{a}\sqrt{a^2-x^2}\,dx$

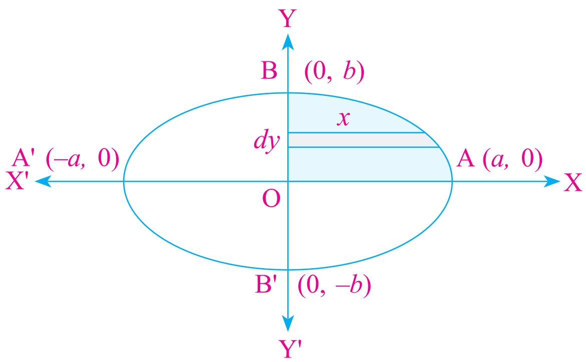
$= \dfrac{4b}{a}\left[\dfrac{x}{2}\sqrt{a^2-x^2}+\dfrac{a^2}{2}sin^{-1}\dfrac{x}{a}\right]_0^a$

$$= \frac{4b}{a}\left[\left(\frac{a}{2}\times 0+\frac{a^2}{2}sin^{-1}1\right)-(0)\right]$$

$$= \frac{4b}{a}\,\frac{a^2}{2}\,\frac{\pi}{2}=\pi ab$$ ચો. એકમ

રીત 2 :

આકૃતિમાં દર્શાવ્યા પ્રમાણે સમક્ષિતિજ પટ્ટીઓ લેતાં, ઉપવલયનું ક્ષેત્રફળ



$$= 4\int_0^b x\,dy$$

$$= \frac{4a}{b}\int_0^b \sqrt{b^2-y^2}\,dy$$

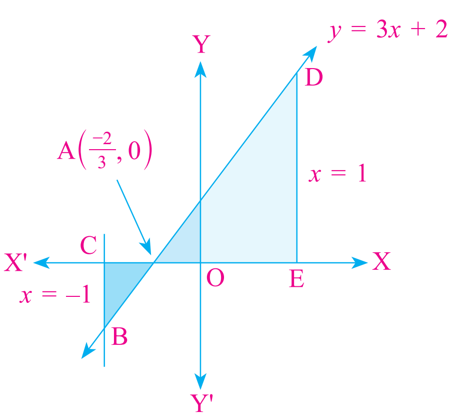
$$= \frac{4a}{b}\left[\frac{y}{2}\sqrt{b^2-y^2}+\frac{b^2}{2}sin^{-1}\frac{y}{b}\right]_0^b$$

$$= \frac{4a}{b}\left[\left(\frac{b}{2}\times 0+\frac{b^2}{2}sin^{-1}(1)\right)-(0)\right]$$

$$= \frac{4a}{b}\,\frac{b^2}{2}\,\frac{\pi}{2}=\pi ab$$ ચો. એકમ

6.

આકૃતિમાં દર્શાવ્યા પ્રમાણે રેખા $y = 3x + 2$, X-અક્ષને $\left(-\frac{2}{3}, 0\right)$માં છેદે છે અને આ આલેખ $x \in \left(-1, -\frac{2}{3}\right)$ માટે X-અક્ષની નીચે છે અને આલેખ $x \in \left(-\frac{2}{3}, 1\right)$ માટે X-અક્ષની ઉપર છે.



માંગેલ ક્ષેત્રફળ

= પ્રદેશ ACBAનું ક્ષેત્રફળ + પ્રદેશ ADEAનું ક્ષેત્રફળ

$$= \left|\int_{-1}^{-\frac{2}{3}} (3x+2)\,dx\right| + \int_{-\frac{2}{3}}^{1} (3x+2)\,dx$$

$$= \left|\left(\frac{3}{2}x^2+2x\right)_{-1}^{-\frac{2}{3}}\right| + \left(\frac{3}{2}x^2+2x\right)_{-\frac{2}{3}}^{1}$$

$$= \left|\left(\frac{3}{2}\left(\frac{4}{9}\right)-\frac{4}{3}\right)-\left(\frac{3}{2}(1)+2(-1)\right)\right| + \left(\frac{3}{2}(1)+2(1)\right) - \left(\frac{3}{2}\left(\frac{4}{9}\right)+2\left(-\frac{2}{3}\right)\right)$$

$$= \left|\frac{2}{3}-\frac{4}{3}-\frac{3}{2}+2\right| + \frac{3}{2}+2-\frac{2}{3}+\frac{4}{3}$$

$$= \left|\frac{-2}{3}-\frac{3}{2}+2\right| + \frac{3}{2}+2+\frac{2}{3}$$

$$= \left|\frac{-4-9+12}{6}\right| + \frac{9+12+4}{6}$$

$$= \frac{1}{6}+\frac{25}{6}$$

$$= \frac{26}{6}$$

$$= \frac{13}{3}$$ ચોરસ એકમ

7.

$$\frac{dy}{dx} = \sqrt{4-y^2}$$

$$\therefore \frac{dy}{\sqrt{4-y^2}} = dx$$

→ બંને બાજુ સંકલન કરતાં,

$$\therefore \int \frac{dy}{\sqrt{4-y^2}} = \int dx$$

$$\therefore sin^{-1}\left(\frac{y}{2}\right) = x + c$$

$$\therefore \frac{y}{2} = sin(x+c)$$

$$\therefore y = 2\,sin(x+c);$$

જે આપેલ વિકલ સમીકરણનો વ્યાપક ઉકેલ છે.

8.

$$\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$$

$$\vec{c} = 3\hat{i} + \hat{j}$$

$$\vec{a} + \lambda\vec{b} = 2\hat{i} + 2\hat{j} + 3\hat{k} - \lambda\hat{i} + 2\lambda\hat{j} + \lambda\hat{k}$$

$$= (2-\lambda)\hat{i} + (2+2\lambda)\hat{j} + (3+\lambda)\hat{k}$$

$\rightarrow (\vec{a} + \lambda \vec{b}) \perp \vec{c}$ હોવાથી,

$\therefore \ (\vec{a} + \lambda \vec{b}) \cdot \vec{c} = 0$

$\therefore \ ((2 - \lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (3 + \lambda)\hat{k}) \cdot (3\hat{i} + \hat{j}) = 0$

$\therefore \ 3(2 - \lambda) + 2 + 2\lambda + 0 = 0$

$\therefore \ 6 - 3\lambda + 2 + 2\lambda = 0$

$\therefore \ 8 - \lambda = 0$

$\therefore \ \lambda = 8$

9.

A(2, 3, 4), B(–1, –2, 1), C(5, 8, 7)

$\vec{a} = \overrightarrow{AB}$

$= (-1, -2, 1) - (2, 3, 4)$

$\vec{a} = (-3, -5, -3)$

$= -3\hat{i} - 5\hat{j} - 3\hat{k}$

$\vec{b} = \overrightarrow{BC}$

$= (5, 8, 7) - (-1, -2, 1)$

$= 6\hat{i} + 10\hat{j} + 6\hat{k}$

હવે, $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ -3 & -5 & -3 \\ 6 & 10 & 6 \end{vmatrix}$

$= 0\hat{i} + 0\hat{j} + 0\hat{k}$

$= \vec{0}$

$\therefore$ A, B, C સમરેખ છે.

(જો $\vec{x} \times \vec{y} = \vec{0}$ તો $\vec{x}$ અને $\vec{y}$ સમરેખ થાય)

10.

$\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1} \Rightarrow \frac{x-5}{7} = \frac{y+2}{5} = \frac{z-0}{1}$

L : $\vec{r} = (5\hat{i} - 2\hat{j} + 0\hat{k}) + \lambda(7\hat{i} - 5\hat{j} + \hat{k})$

$\therefore \ \vec{b_1} = 7\hat{i} - 5\hat{j} + \hat{k}$

$\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$

M : $\vec{r} = (0\hat{i} + 0\hat{j} + 0\hat{k}) + \mu(\hat{i} + 2\hat{j} + 3\hat{k})$

$\therefore \ \vec{b_2} = \hat{i} + 2\hat{j} + 3\hat{k}$

હવે, $\vec{b_1} \cdot \vec{b_2}$

$= (7\hat{i} - 5\hat{j} + \hat{k}) \cdot (\hat{i} + 2\hat{j} + 3\hat{k})$

$= 7 - 10 + 3$

$= 0$

$\therefore$ L અને M પરસ્પર લંબ છે.

11.

ધારો કે ઘટના E એ યાદૃચ્છિક રીતે પસંદ થયેલ વિદ્યાર્થી ધોરણ XIIમાં અભ્યાસ કરે છે તે દર્શાવે છે અને ઘટના F એ યાદૃચ્છિક રીતે પસંદ થયેલ વિદ્યાર્થી છોકરી છે તે દર્શાવે છે. આપણે P(E | F) શોધવાનું છે.

હવે, $P(F) = \frac{430}{1000}$

$= 0.43$

અને $P(E \cap F) = \frac{43}{1000}$

$= 0.043$

તેથી, $P(E \mid F) = \frac{P(E \cap F)}{P(F)}$

$= \frac{0.043}{0.43}$

$= 1$

12.

પાસાને ત્રણ વખત ફેંકતા મળતાં પરિણામો

$n = 216$

ઘટના E : ત્રીજી વખત ફેંકતા 4 મળે છે.

E = {(1, 1, 4), (1, 2, 4), (1, 3, 4), (1, 4, 4), (1, 5, 4), (1, 6, 4), (2, 1, 4), (2, 2, 4), (2, 3, 4), (2, 4, 4), (2, 5, 4), (2, 6, 4), (3, 1, 4), (3, 2, 4), (3, 3, 4), (3, 4, 4), (3, 5, 4), (3, 6, 4), (4, 1, 4), (4, 2, 4), (4, 3, 4), (4, 4, 4), (4, 5, 4), (4, 6, 4), (5, 1, 4), (5, 2, 4), (5, 3, 4), (5, 4, 4), (5, 5, 4), (5, 6, 4), (6, 1, 4), (6, 2, 4), (6, 3, 4), (6, 4, 4), (6, 5, 4), (6, 6, 4)}

$\therefore \ r = 36$

$\therefore \ P(E) = \frac{r}{n}$

$= \frac{36}{216}$

$= \frac{1}{6}$

ઘટના F : પ્રથમ બે વખત ફેંકતા 6 અને 5 મળે.

F = {(6, 5, 1), (6, 5, 2), (6, 5, 3), (6, 5, 4), (6, 5, 5), (6, 5, 6)}

$\therefore \ r = 6$

$\therefore \ P(F) = \frac{r}{n}$

$= \frac{6}{216}$

$= \frac{1}{36}$

$\therefore \ E \cap F = \{(6, 5, 4)\}$

$\therefore \ r = 1$

$\therefore\ P(E \cap F) = \dfrac{1}{216}$

$\therefore\ P(E \mid F) = \dfrac{P(E \cap F)}{P(F)}$

$= \dfrac{\frac{1}{216}}{\frac{1}{36}}$

$= \dfrac{1}{6}$

વિભાગ-B

13.

અહીં $f : R \to R,\ f(x) = |x|$

$x_1 = -1$ લેતાં, $f(-1) = |-1| = 1$

$x_2 = 1$ લેતાં, $f(1) = |1| = 1$

$x_1 \neq x_2$ પરંતુ $f(x_1) = f(x_2)$

$\therefore f$ એ એક-એક વિધેય નથી.

$\forall\ x \in R$, આપણે જાણીએ છીએ કે, $|x| \geq 0$

$\therefore f(x) \geq 0$

$\therefore f$ નો વિસ્તાર $= [o, \infty) = R^+ \cup \{0\} \neq$ સહપ્રદેશ (R)

$\therefore f$ એ વ્યાપ્ત વિધેય નથી.

14.

$2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$

$4X + 6Y = \begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix}$ .... (1)

($\because$ 2 વડે ગુણતાં)

$3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$

3 વડે ગુણતાં,

$9X + 6Y = \begin{bmatrix} 6 & -6 \\ -3 & 15 \end{bmatrix}$ .... (2)

પરિણામ (2)માંથી (1) બાદ કરતાં,

$9X + 6Y = \begin{bmatrix} 6 & -6 \\ -3 & 15 \end{bmatrix}$

$4X + 6Y = \begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix}$

$-\quad -\quad -$

$5X = \begin{bmatrix} 6 & -6 \\ -3 & 15 \end{bmatrix} - \begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix}$

$\therefore\ 5X = \begin{bmatrix} 2 & -12 \\ -11 & 15 \end{bmatrix}$

$\therefore\ X = \dfrac{1}{5}\begin{bmatrix} 2 & -12 \\ -11 & 15 \end{bmatrix}$

$X = \begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix}$

$X = \begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix}$ કિંમત

$3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$ માં મૂકતાં,

$3\begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix} + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$

$\begin{bmatrix} \frac{6}{5} & -\frac{36}{5} \\ -\frac{33}{5} & 9 \end{bmatrix} + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$

$\therefore\ 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix} - \begin{bmatrix} \frac{6}{5} & -\frac{36}{5} \\ -\frac{33}{5} & 9 \end{bmatrix}$

$\therefore\ 2Y = \begin{bmatrix} 2-\frac{6}{5} & -2+\frac{36}{5} \\ -1+\frac{33}{5} & 5-9 \end{bmatrix} = \begin{bmatrix} \frac{4}{5} & \frac{26}{5} \\ \frac{28}{5} & -4 \end{bmatrix}$

$\therefore\ Y = \dfrac{1}{2}\begin{bmatrix} \frac{4}{5} & \frac{26}{5} \\ \frac{28}{5} & -4 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$

આમ, $X = \begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix}$, તથા $Y = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$

15.

આપણને $AB = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 5 & -14 \end{bmatrix}$ મળે.

$|AB| = -11 \neq 0$ હોવાથી, $(AB)^{-1}$ નું અસ્તિત્વ છે અને

$(AB)^{-1} = \dfrac{1}{|AB|}\ adj(AB)$

$= -\dfrac{1}{11}\begin{bmatrix} -14 & -5 \\ -5 & -1 \end{bmatrix}$

$= \dfrac{1}{11}\begin{bmatrix} 14 & 5 \\ 5 & 1 \end{bmatrix}$ મળે છે.

વળી, $|A| = -11 \neq 0$ અને $|B| = 1 \neq 0$.

આથી A^{-1} અને B^{-1} બંનેનું અસ્તિત્વ છે.

અને $A^{-1} = -\dfrac{1}{11}\begin{bmatrix} -4 & -3 \\ -1 & 2 \end{bmatrix}$, $B^{-1} = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$

માટે $B^{-1}A^{-1} = -\dfrac{1}{11}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -4 & -3 \\ -1 & 2 \end{bmatrix}$

$= -\dfrac{1}{11}\begin{bmatrix} -14 & -5 \\ -5 & -1 \end{bmatrix}$

$= \dfrac{1}{11}\begin{bmatrix} 14 & 5 \\ 5 & 1 \end{bmatrix}$

તેથી $(AB)^{-1} = B^{-1}A^{-1}$

16.

ધારો કે, $u = \left(x+\frac{1}{x}\right)^x$ અને $v = x^{\left(1+\frac{1}{x}\right)}$

$\therefore y = u + v$

હવે, બંને બાજુ x પ્રત્યે વિકલન કરતાં,

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \text{.........} (1)$$

અહીં, $u = \left(x+\frac{1}{x}\right)^x$ ની

બંને બાજુ log લેતાં,

$$log\ u = x\ log\left(x+\frac{1}{x}\right)$$

હવે, બંને બાજુ x પ્રત્યે વિકલન કરતાં,

$$\therefore \frac{1}{u}\frac{du}{dx} = x\frac{d}{dx}log\left(x+\frac{1}{x}\right) + log\left(x+\frac{1}{x}\right)\frac{d}{dx}x$$

$$\therefore \frac{1}{u}\frac{du}{dx} = \frac{x}{\left(x+\frac{1}{x}\right)}\frac{d}{dx}\left(x+\frac{1}{x}\right) + log\left(x+\frac{1}{x}\right)$$

$$= \frac{x^2}{x^2+1}\left(1-\frac{1}{x^2}\right) + log\left(x+\frac{1}{x}\right)$$

$$= \frac{x^2}{x^2+1}\left(\frac{x^2-1}{x^2}\right) + log\left(x+\frac{1}{x}\right)$$

$$\therefore \frac{1}{u}\frac{du}{dx} = \frac{x^2-1}{x^2+1} + log\left(x+\frac{1}{x}\right)$$

$$\therefore \frac{du}{dx} = u\left(\frac{x^2-1}{x^2+1} + log\left(x+\frac{1}{x}\right)\right)$$

$$\therefore \frac{du}{dx} = \left(x+\frac{1}{x}\right)^x\left[\frac{x^2-1}{x^2+1} + log\left(x+\frac{1}{x}\right)\right] \quad \text{...} (2)$$

હવે, $v = x^{\left(1+\frac{1}{x}\right)}$ ની

બંને બાજુ log લેતાં,

$$log\ v = \left(1+\frac{1}{x}\right) log\ x$$

હવે, બંને બાજુ x પ્રત્યે વિકલન કરતાં,

$$\frac{1}{v}\frac{dv}{dx} = \left(1+\frac{1}{x}\right)\frac{d}{dx}\ logx + logx\ \frac{d}{dx}\left(1+\frac{1}{x}\right)$$

$$\therefore \frac{1}{v}\frac{dv}{dx} = \left(1+\frac{1}{x}\right)\frac{1}{x} + log\,x\left(0-\frac{1}{x^2}\right)$$

$$= \frac{x+1}{x^2} - \frac{logx}{x^2}$$

$$\therefore \frac{dv}{dx} = v\left(\frac{x+1-logx}{x^2}\right)$$

$$\therefore \frac{dv}{dx} = x^{\left(1+\frac{1}{x}\right)}\left(\frac{x+1-logx}{x^2}\right) \quad \text{......} (3)$$

પરિણામ (1) માં પરિણામ (2) અને (3) ની કિંમત મૂકતાં,

$$\frac{dy}{dx} = \left(x+\frac{1}{x}\right)^x\left[\frac{x^2-1}{x^2+1} + \log\left(x+\frac{1}{x}\right)\right] + x^{\left(1+\frac{1}{x}\right)}\left[\frac{x+1-logx}{x^2}\right]$$

17.

અહીં, x અને y બંને ધન છે.

$x + y = 35 \quad (x < 35, y < 35)$

$\therefore x = 35 - y$

$x^2y^5 = (35-y)^2 y^5$

$\therefore f(y) = (35-y)^2 y^5$

$\therefore f'(y) = 5y^4 \cdot (35-y)^2 + y^5 \cdot 2(35-y)(-1)$

$\therefore f'(y) = 5y^4 \cdot (35-y)^2 - 2y^5(35-y)$

$\therefore f'(y) = (35-y)\, y^4\, (5(35-y) - 2y)$

$= (35-y)\, y^4\, (175 - 5y - 2y)$

$= (35-y)\, y^4\, (175 - 7y)$

$\therefore f'(y) = 7y^4(35-y)(25-y)$

$\rightarrow f'(y) = 0$

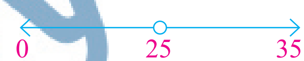
$\therefore 7y^4(35-y)(25-y) = 0$

$\therefore y = 0$ કે $35 - y = 0$ કે $25 - y = 0$

$\therefore y = 0$ કે $y = 35$ કે $y = 25$

$y = 0, 35$ શક્ય નથી. $\quad (\because y \neq 0, y < 35)$

$\therefore y = 25$



$y < 25, f'(y) > 0$

$y > 25, f'(y) < 0$

$\therefore y$ ને $x = 25$ આગળ મહત્તમ મૂલ્ય મળે.

$\therefore$ એક સંખ્યા $y = 25$

બીજી સંખ્યા $x = 10$

18.

$\vec{a} = \hat{i} + \hat{j} + \hat{k}$

$\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$

$\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$

$2\vec{a} - \vec{b} + 3\vec{c}$

$= 2\hat{i} + 2\hat{j} + 2\hat{k} - 2\hat{i} + \hat{j} - 3\hat{k} + 3\hat{i} - 6\hat{j} + 3\hat{k}$

$= 3\hat{i} - 3\hat{j} + 2\hat{k}$

$2\vec{a} - \vec{b} + 3\vec{c}$ ને સમાંતર સદિશ,

$$= \frac{2\vec{a} - \vec{b} + 2\vec{c}}{|2\vec{a} - \vec{b} + 3\vec{c}\,|}$$

$$= \frac{3\hat{i} - 3\hat{j} + 2\hat{k}}{\sqrt{9+9+4}}$$

$$= \frac{3}{\sqrt{22}}\hat{i} - \frac{3}{\sqrt{22}}\hat{j} + \frac{2}{\sqrt{22}}\hat{k}$$

19.

બે રેખાઓ સમાંતર છે.

આપણી પાસે $\vec{a_1} = \hat{i} + 2\hat{j} - 4\hat{k}$,

$\vec{a_2} = 3\hat{i} + 3\hat{j} - 5\hat{k}$ અને

$\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ છે.

આથી, રેખાઓ વચ્ચેનું અંતર

$$d = \left| \frac{\vec{b} \times (\vec{a_2} - \vec{a_1})}{|\vec{b}|} \right|$$

$$= \left| \frac{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 2 & 1 & -1 \end{vmatrix}}{\sqrt{4+9+36}} \right|$$

$$= \frac{|-9\hat{i} + 14\hat{j} - 4\hat{k}|}{\sqrt{49}}$$

$$= \frac{\sqrt{293}}{\sqrt{49}}$$

$$= \frac{\sqrt{293}}{7} \text{ એકમ}$$

20.

$x \geq 3$

$x + y \geq 5$

$x + 2y \geq 6$

$y \geq 0$

હેતુલક્ષી વિધેય $Z = -x + 2y$

$x = 3$ ... (i)

$x + y = 5$ ... (ii)

x	0	5
y	5	0

(0, 5) ×
(5, 0) ×

$x + 2y = 6$ ... (iii)

x	0	6
y	3	0

(0, 3) ×
(6, 0) ✓

(i) અને (ii) નો લોપ,

$\therefore y = 5 - 3 = 2$ $\quad \therefore$ (3, 2) ✓

(ii) અને (iii) નો લોપ,

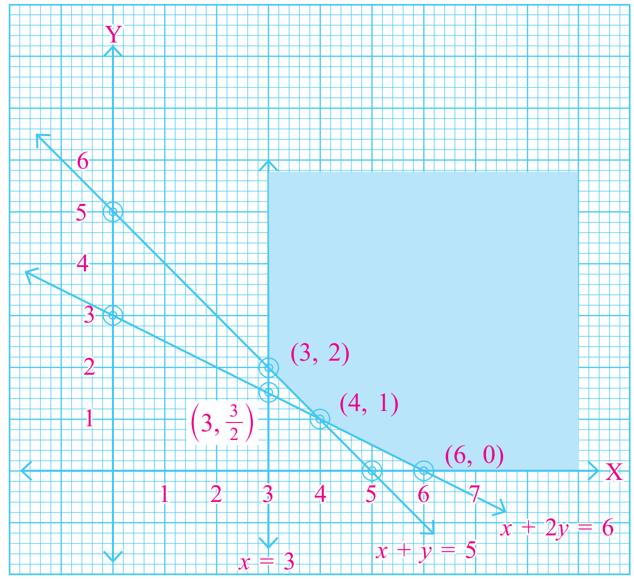
$$\begin{array}{r} x + y = 5 \\ -\;x + 2y = 6 \\ \hline y = 1 \end{array}$$

$\therefore x = 4$ (4, 1) ✓

(i) અને (iii) નો લોપ,

$2y = 3$

$\therefore y = \frac{3}{2}$ $\quad \left(3, \frac{3}{2}\right)$ ×



આકૃતિમાં આપેલ અસમતાઓનો આલેખ દર્શાવ્યો છે જે અસિમિત છે. શક્ય ઉકેલ પ્રદેશના શિરોબિંદુઓ (3, 2), (4, 1) અને (6, 0) મળે.

શક્ય ઉકેલ પ્રદેશના શિરોબિંદુ	$Z = -x + 2y$
(3, 2)	1 ← મહત્તમ
(4, 1)	–2
(6, 0)	–6

$-x + 2y \leq 1$

અસંમિત પ્રદેશમાંથી (6, 4) લઈને ચકાસતા,

$\therefore -6 + 8 \leq 1$

$\therefore 2 \leq 1$

$\therefore z$ ને મહત્તમ મૂલ્ય ન મળે.

21.

ઘટના E_1 : પ્રથમ સમૂહ જીતે.

ઘટના E_2 : બીજો સમૂહ જીતે.

ઘટના A : જીતેલા સમૂહ દ્વારા વસ્તુ રજૂ થાય છે.

નવી ઉત્પાદિત વસ્તુ સમૂહ દ્વિતીય દ્વારા રજૂ થાય તેની સંભાવના,

અહીં, $P(E_1) = 0.6$ તથા $P(A | E_1) = 0.7$

$P(E_2) = 0.4$ તથા $P(A | E_2) = 0.3$

$\therefore$ $P(A) = P(E_1) \cdot P(A | E_1) + P(E_2) \cdot P(A | E_2)$

$= 0.6 \times 0.7 + 0.4 \times 0.3$

$= 0.42 + 0.12$

$= 0.54$

$\therefore \quad P(E_2 \mid A) = \dfrac{P(E_2) \cdot P(A \mid E_2)}{P(A)}$

$= \dfrac{0.4 \times 0.3}{0.54}$

$= \dfrac{0.12}{0.54}$

$= \dfrac{12}{54}$

$\therefore \quad P(E_2 \mid A) = \dfrac{2}{9}$

વિભાગ-C

22.

અહીં A અને B સંમિત શ્રેણિક છે.

$\therefore$ A' = A તથા B' = B ... (1)

હવે, X = AB – BA લેતાં

X' = (AB – BA)'

= (AB)' – (BA)'

= B'A' – (A'B')

= BA – AB ($\because$ પરિણામ (1))

= – (AB – BA)

= – X

$\therefore$ X એ વિસંમિત શ્રેણિક છે.

$\therefore$ AB – BA એ વિસંમિત શ્રેણિક છે.

23.

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -2-9+12 & 0-2+2 & 1+3-4 \\ 0+18-18 & 0+4-3 & 0-6+6 \\ -6-18+24 & 0-4+4 & 3+6-8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

આથી, $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$

હવે, આપેલ સમીકરણ સંહતિને શ્રેણિક સ્વરૂપમાં નીચે પ્રમાણે લખી શકાય :

AX = B

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

અથવા $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}^{-1}\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

$$= \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2+0+2 \\ 9+2-6 \\ 6+1-4 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$$

તેથી $x = 0$, $y = 5$ અને $z = 3$.

24.

રીત-1 :

$y = sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$

$\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-\left(\dfrac{2x}{1+x^2}\right)^2}} \cdot \dfrac{d}{dx}\left(\dfrac{2x}{1+x^2}\right)$

$= \dfrac{1}{\sqrt{1-\dfrac{4x^2}{(1+x^2)^2}}} \times \dfrac{(1+x^2)\frac{d}{dx}2x - 2x\frac{d}{dx}(1+x^2)}{(1+x^2)^2}$

$= \dfrac{(1+x^2)}{\sqrt{(1+x^2)^2-4x^2}} \times \dfrac{(1+x^2)(2)-(2x)(2x)}{(1+x^2)^2}$

$= \dfrac{1}{\sqrt{(1+x^2)^2-4x^2}} \times \dfrac{2+2x^2-4x^2}{(1+x^2)}$

$= \dfrac{1}{\sqrt{1+2x^2+x^4-4x^2}} \times \dfrac{2-2x^2}{1+x^2}$

$= \dfrac{2(1-x^2)}{\sqrt{1-2x^2+x^4}} \times \dfrac{1}{1+x^2}$

$= \dfrac{2(1-x^2)}{\sqrt{(1-x^2)^2}} \times \dfrac{1}{1+x^2}$

$= \dfrac{2(1-x^2)}{|1-x^2|} \times \dfrac{1}{1+x^2}$ .... (1)

વિકલ્પ-1 : $|x| < 1$

$\therefore \; -1 < x < 1$

$\therefore \; 0 < x^2 < 1$

$\therefore \; 0 < 1 - x^2$

$\therefore \; |1 - x^2| = 1 - x^2$

$\dfrac{dy}{dx} = \dfrac{2(1-x^2)}{(1-x^2)} \times \dfrac{1}{1+x^2}$

$\therefore \; \dfrac{dy}{dx} = \dfrac{2}{1+x^2}$

વિકલ્પ-2 : $|x| > 1$

$\therefore \; x < -1$ અને $x > 1$

$\therefore \; x^2 > 1$

$\therefore \; 1 - x^2 < 0$

$\therefore \; |1 - x^2| = -(1 - x^2)$

$$\frac{dy}{dx} = \frac{2(1-x^2)}{-(1-x^2)} \times \frac{1}{x^2-1}$$

$$\therefore \quad \frac{dy}{dx} = \frac{-2}{1+x^2}$$

વિકલ્પ-3 : $x = \pm 1$ લેતાં, જે કોઈ અંતરાલ નથી.

$\therefore \quad \frac{dy}{dx}$ મળે નહીં.

રીત-2 :

$$y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

ધારો કે, $x = \tan\theta \qquad \theta \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

$\theta = \tan^{-1}x$

હવે, $y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$y = \sin^{-1}(\sin 2\theta)$ .... (1)

વિકલ્પ-1 : $-1 < x < 1$

$\tan\left(\frac{-\pi}{4}\right) < \tan\theta < \tan\frac{\pi}{4}$

$\frac{-\pi}{4} < \theta < \frac{\pi}{4}$

$\frac{-\pi}{2} < 2\theta < \frac{\pi}{2}$

$2\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \subset \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ ... (1)

$\therefore \ y = \sin^{-1}(\sin 2\theta)$

$= 2\theta$ ($\because$ પરિણામ (1) પરથી)

$\therefore \ y = 2\tan^{-1}x$

$$\frac{dy}{dx} = 2 \cdot \frac{d}{dx}\tan^{-1}x$$

$$\therefore \quad \frac{dy}{dx} = \frac{2}{1+x^2}$$

વિકલ્પ-2 : $x > 1$

$\therefore \ \tan\theta > \tan\frac{\pi}{4}$

$\therefore \ \frac{\pi}{4} < \theta < \frac{\pi}{2}$

$\therefore \ \frac{\pi}{2} < 2\theta < \pi$

$\therefore \ \frac{\pi}{2} - \pi < 2\theta - \pi < 0$

$\therefore \ \frac{-\pi}{2} < 2\theta - \pi < 0$

$(2\theta - \pi) \in \left(-\frac{\pi}{2}, 0\right) \subset \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ ... (2)

હવે, $-\sin(2\theta - \pi)$

$= \sin(\pi - 2\theta)$

$= \sin 2\theta$

$\therefore y = \sin^{-1}(\sin 2\theta)$

$= \sin^{-1}(-\sin(2\theta - \pi))$

$= -\sin^{-1}(\sin(2\theta - \pi))$

$= -(2\theta - \pi)$ ($\because$ પરિણામ (2) પરથી)

$\therefore y = \pi - 2\theta$

$= \pi - 2\tan^{-1}x$

$= \pi - 2\tan^{-1}x$

x પ્રત્યે વિકલન કરતાં,

$$\frac{dy}{dx} = -2\frac{d}{dx}\tan^{-1}x$$

$$\therefore \quad \frac{dy}{dx} = \frac{-2}{1+x^2}$$

વિકલ્પ-3 : $x < -1$

$-\infty < x < -1$

$\therefore \ \tan\left(\frac{-\pi}{2}\right) < \tan\theta < \tan\left(\frac{-\pi}{4}\right)$

$\therefore \ \frac{-\pi}{2} < \theta < \frac{-\pi}{4}$

$\therefore \ -\pi < 2\theta < \frac{-\pi}{2}$

$\therefore \ 0 < 2\theta + \pi < \frac{-\pi}{2} + \pi$

$\therefore \ 0 < 2\theta + \pi < \frac{\pi}{2}$

$(2\theta + \pi) \in \left(0, \frac{\pi}{2}\right) \subset \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ ... (3)

હવે, $-\sin(2\theta + \pi)$

$= \sin 2\theta$

$y = \sin^{-1}(\sin 2\theta)$

$= \sin^{-1}(-\sin(2\theta + \pi))$

$= -\sin^{-1}(\sin(2\theta + \pi))$

$= -(2\theta + \pi)$ ($\because$ પરિણામ (3) પરથી)

$= -2\theta - \pi$

$y = -2\tan^{-1}x - \pi$

$$\therefore \quad \frac{dy}{dx} = \frac{-2}{1+x^2}$$

વિકલ્પ-4 : $x = \pm 1$

જે કોઈ અંતરાલ નથી.

$\therefore \quad \frac{dy}{dx}$ મળે નહીં.

OR

રીત-3 :

$$y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$= 2\tan^{-1}x, \ \forall\, x \in R$

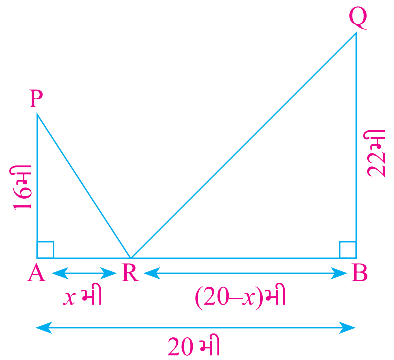
$$\therefore \quad \frac{dy}{dx} = 2 \cdot \frac{d}{dx}\tan^{-1}x = \frac{2}{1+x^2}$$

25.

ધારો કે R એ AB પર માંગ્યા પ્રમાણેનું બિંદુ છે.

AR = x મીટર

$\therefore$ RB = $(20 - x)$ મીટર ($\because$ AB = 20 મીટર)



આકૃતિ પરથી, $RP^2 = AR^2 + AP^2$

અને $RQ^2 = RB^2 + BQ^2$

$\therefore RP^2 + RQ^2 = AR^2 + AP^2 + RB^2 + BQ^2$

$= x^2 + (16)^2 + (20 - x)^2 + (22)^2$

$= 2x^2 - 40x + 1140$

ધારો કે $S \equiv S(x)$

$= RP^2 + RQ^2$

$= 2x^2 - 40x + 1140$

$\therefore S'(x) = 4x - 40$

હવે, $S'(x) = 0$ લેતાં, $x = 10$ મળે.

તેમજ $S''(x) = 4 > 0, \forall x$

અને તેથી $S''(10) > 0$

આથી, દ્વિતીય વિકલિત કસોટી પરથી,

$x = 10$ આગળ S ને સ્થાનીય ન્યૂનતમ મૂલ્ય છે.

આથી, $RP^2 + RQ^2$ ન્યૂનતમ બને તે માટે

$\overline{AB}$ પરના બિંદુ R નું બિંદુ A થી અંતર

AR = x = 10 મીટર.

26.

$$I = \int tan^{-1}\sqrt{\frac{1-x}{1+x}}\, dx$$

અહીં, $x = cos\,\theta$ આદેશ લેતાં,

$\therefore dx = -sin\,\theta\, d\theta$

$$I = \int tan^{-1}\sqrt{\frac{1-cos\,\theta}{1+cos\,\theta}}\,(-sin\,\theta)\, d\theta$$

$$I = -\int tan^{-1}\sqrt{tan^2\frac{\theta}{2}}\, sin\,\theta\, d\theta$$

$$= -\int tan^{-1}\left(tan\frac{\theta}{2}\right) sin\,\theta\, d\theta$$

$$= -\int \frac{\theta}{2}\cdot sin\,\theta\, d\theta$$

$$I = \frac{-1}{2}\int \theta\cdot sin\,\theta\, d\theta$$

$$I = \frac{-1}{2} I_1 \quad ... (1)$$

$$I_1 = \int \theta\cdot sin\,\theta\, d\theta$$

$\rightarrow$ હવે, $u = \theta$; $v = sin\,\theta$

$\frac{du}{d\theta} = 1$

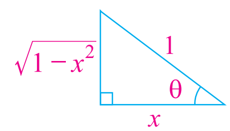
ખંડશઃ સંકલન કરતાં,

$$I_1 = \theta\int sin\,\theta\, d\theta - \int\left(1\int sin\,\theta\, d\theta\right) d\theta$$

$$= -\theta\cdot cos\,\theta + \int cos\,\theta\, d\theta$$

$$I_1 = -\theta\, cos\,\theta + sin\,\theta + c$$

$\rightarrow$ હવે, $x = cos\,\theta$



$\theta = cos^{-1}x$

$\sqrt{1-x^2} = sin\,\theta$

$$I_1 = -cos^{-1}x\cdot x + \sqrt{1-x^2} + c_1$$

I_1 ની કિંમત પરિણામ (1) માં મૂકતાં,

$$I = \frac{-1}{2}\left[-x\cdot cos^{-1}x + \sqrt{1-x^2} + c_1\right] + c'$$

$$I = \frac{1}{2}\left[x\cdot cos^{-1}x - \sqrt{1-x^2}\right] + c$$

$$\left(\because \frac{-1}{2}c_1 + c' = c\right)$$

27.

રીત 1 :

$(x^3 - 3xy^2)\, dx = (y^3 - 3x^2y)\, dy$

$$\therefore \frac{dy}{dx} = \frac{x^3 - 3xy^2}{y^3 - 3x^2y}$$

$$\therefore \frac{dy}{dx} = \frac{1 - 3\left(\frac{y}{x}\right)^2}{\left(\frac{y}{x}\right)^3 - 3\left(\frac{y}{x}\right)} \quad ... (1)$$

$\frac{y}{x} = v$ આદેશ લેતાં,

$\therefore y = vx$

$\rightarrow$ x પ્રત્યે વિકલન કરતાં,

$$\therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$\rightarrow$ આ કિંમતો પરિણામ (1) માં મૂકતાં,

$$\therefore v + x\frac{dv}{dx} = \frac{1-3v^2}{v^3-3v}$$

$$\therefore x\frac{dv}{dx} = \frac{1-3v^2}{v^3-3v} - v$$

$$\therefore \quad x\frac{dv}{dx} = \frac{1-3v^2-v^4+3v^2}{v^3-3v}$$

$$\therefore \quad x\frac{dv}{dx} = \frac{1-v^4}{v^3-3v}$$

$$\therefore \quad \left(\frac{v^3-3v}{1-v^4}\right) dv = \frac{dx}{x}$$

→ બંને બાજુ સંકલન કરતાં,

$$\therefore \int \frac{v^3}{1-v^4}\, dv - 3\int \frac{v\, dv}{1-v^4} = \int \frac{dx}{x}$$

$$\therefore -\frac{1}{4}\int \frac{-4v^3}{1-v^4}\, dv + 3\int \frac{v}{v^4-1} dv = \int \frac{dx}{x}$$

$$\therefore -\frac{1}{4}\int \frac{-4v^3}{(1-v^4)}\, dv + 3\int \frac{v\ dv}{(v^2)^2-1} = \int \frac{dx}{x}$$

→ બીજા સંકલનમાં $v^2 = t$ આદેશ લેતાં,

$$\therefore \quad 2v\ dv = dt$$

$$\therefore \quad v\ dv = \frac{dt}{2}$$

$$\therefore -\frac{1}{4}\int \frac{\frac{d}{dv}(1-v^4)}{(1-v^4)}\, dv + \frac{3}{2}\int \frac{dt}{t^2-1} = \int \frac{dx}{x}$$

$$\therefore -\frac{1}{4}\log|1-v^4| + \frac{3}{2}\cdot\frac{1}{2}\log\left|\frac{t-1}{t+1}\right| = \log|x| + \log|c'|$$

$$\therefore -\frac{1}{4}\log|1-v^4| + \frac{3}{4}\log\left|\frac{v^2-1}{v^2+1}\right| = \log|xc'|$$

$$\therefore \frac{1}{4}\log\left|\frac{1}{1-v^4}\right| + \frac{3}{4}\log\left|\frac{v^2-1}{v^2+1}\right| = \log|c'x|$$

$$\therefore \log\left|\left(\frac{1}{1-v^4}\right)^{\frac{1}{4}}\right| + \log\left|\left(\frac{v^2-1}{v^2+1}\right)^{\frac{3}{4}}\right| = \log|c'x|$$

$$\therefore \log\left|\frac{1}{(1-v^4)^{\frac{1}{4}}} \times \frac{(v^2-1)^{\frac{3}{4}}}{(v^2+1)^{\frac{3}{4}}}\right| = \log|c'x|$$

$$\therefore \frac{(v^2-1)^{\frac{3}{4}}}{(v^4-1)^{\frac{1}{4}}} \times \frac{1}{(v^2+1)^{\frac{3}{4}}} = c'x$$

$$\therefore \frac{(v^2-1)^{\frac{3}{4}}}{(v^2-1)^{\frac{1}{4}}(v^2+1)^{\frac{1}{4}}(v^2+1)^{\frac{3}{4}}} = c'x$$

$$\therefore \frac{(v^2-1)^{\frac{1}{2}}}{v^2+1} = c'x$$

→ $v = \frac{y}{x}$ પરથી,

$$\therefore \frac{\left[\left(\frac{y}{x}\right)^2-1\right]^{\frac{1}{2}}}{\left(\frac{y}{x}\right)^2+1} = c'x$$

$$\therefore \frac{[y^2-x^2]^{\frac{1}{2}}}{x} \times \frac{x^2}{y^2+x^2} = c'x$$

$$\therefore (y^2-x^2)^{\frac{1}{2}} = c'(x^2+y^2)$$

$$\therefore (y^2-x^2) = (c')^2[x^2+y^2]^2$$

$$\therefore (x^2-y^2) = -(c')^2[x^2+y^2]^2$$

$$\therefore (x^2-y^2) = c[x^2+y^2] \quad (\because -(c')^2 = c)$$

રીત 2 :

$$x^2 - y^2 = c(x^2+y^2)$$

$$\therefore \frac{x^2-y^2}{(x^2+y^2)^2} = c$$

x ની સાપેક્ષે વિકલન કરતાં,

$$\frac{d}{dx}\left[\frac{x^2-y^2}{(x^2+y^2)^2}\right] = 0$$

$$\therefore (x^2+y^2)^2\left(2x-2y\frac{dy}{dx}\right) - (x^2-y^2)\cdot 2(x^2+y^2)\left(2x+2y\frac{dy}{dx}\right) = 0$$

$$\therefore 2(x^2+y^2)\left[(x^2+y^2)\left(x-y\frac{dy}{dx}\right) - (2x^2-2y^2)\left(x+y\frac{dy}{dx}\right)\right] = 0$$

$$\therefore x^3 - x^2y\frac{dy}{dx} + xy^2 - y^3\frac{dy}{dx} - 2x^3 - 2x^2y\cdot\frac{dy}{dx} + 2xy^2 + 2y^3\frac{dy}{dx} = 0$$

$$\therefore y^3\frac{dy}{dx} - 3x^2y\frac{dy}{dx} - x^3 + 3xy^2 = 0$$

$$\therefore \frac{dy}{dx}(y^3 - 3x^2y) = x^3 - 3xy^2$$

$$\therefore (y^3 - 3x^2y)\, dy = (x^3 - 3xy^2)\, dx$$

$$\therefore (x^3 - 3xy^2)\, dy - (y^3 - 3x^2y)\, dy = 0$$

જે આપેલ વિકલ સમીકરણનો વ્યાપક ઉકેલ છે.